



Coupled Frequency-Dependent and Ionization-Dependent Tower Footing Response and the Seasonal Weighting of Back flashover Rate for a 150 kV Double-Circuit Transmission Line in Continental Greece

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Abstract: Design practice for the lightning performance of transmission lines represents the tower footing either by a constant resistance, by a current-dependent resistance accounting for soil ionization, or by a frequency-dependent (FD) soil model, but rarely by the two dispersive mechanisms together. This paper computes the coupled FD and ionization response of the standard footing of the Hellenic 150 kV double-circuit Series-4 tower and propagates it to the backflashover rate (BFR) of a 42 km line in the karst of Sterea Ellada. Three results are reported. Omitting either mechanism is a first-order error: a constant-resistance footing overestimates the impulse impedance by up to 344 % and the BFR by up to 185 %, while an FD-only or an ionization-only representation still overestimates BFR by up to 51 % and 109 % respectively. Within a transmission-line representation of the electrode the two mechanisms are almost exactly multiplicatively separable, because ionization acts on the geometric factor of the electrode while frequency dependence acts on the propagation constant of the soil, so two independent correction factors suffice. Because BFR is a convex function of low-frequency resistivity, weighting the seasonal resistivity of Mediterranean karst by the observed bimodal Greek lightning climatology raises the computed BFR by 43 % to 267 % relative to the conventional annual-average calculation, which is therefore systematically non-conservative. Revised effective counterpoise lengths for the standard 10 mm steel counterpoise are given as a design chart.

Keywords: Lightning protection; backflashover rate; frequency-dependent soil parameters; soil ionization; tower footing impedance; transmission lines.

I. INTRODUCTION

The backflashover rate of a shielded overhead transmission line is governed, more than by any other single quantity, by the transient impedance of the tower footing. Conventional practice represents that impedance by its low-frequency resistance, optionally reduced by a current-dependent factor accounting for soil ionization in the high-field region surrounding the electrode^{1,2}. Over the last decade a second dispersive mechanism has been established: the conductivity and permittivity of soil are themselves strongly frequency dependent over the spectral range occupied by a lightning current. The empirical model of Alipio and Visacro³, derived from an extensive campaign of field measurements, is recommended by CIGRE Working Group C4.33⁴, which advises that frequency dependence may be neglected below 300 $\Omega \cdot m$, should be considered between 300 and 700 $\Omega \cdot m$, and must be included above 700 $\Omega \cdot m$.

The two mechanisms have, however, been studied almost entirely in isolation. Studies adopting the frequency-dependent (FD) soil model generally treat the electrode as linear and omit ionization⁵; studies modelling ionization generally retain constant soil parameters². This is uncomfortable, because both mechanisms reduce the transient impedance, both are strongest in exactly the high-resistivity soils where backflashover is design-limiting, and there is no a priori reason to expect their combined effect to be the sum, or the product, of their separate effects. Ionization enlarges the effective electrode and so alters the geometry that the dispersive soil surrounds; frequency dependence lowers the effective resistivity at high frequency and so alters the field that drives ionization.

A second and quite separate simplification is universal in the literature: the low-frequency resistivity ρ_0 is taken as a single annual value. In Mediterranean karst this is a poor assumption. The thin terra rossa fill of fissured limestone drains rapidly, and its resistivity varies by close to an order of magnitude between the saturated winter state and the summer drought. More importantly, lightning in Greece is not uniformly distributed through the year: measurements from the Hellenic precision lightning network show that autumn accounts for 43.5 % and summer for 33.8 % of annual cloud-to-ground

flashes⁶, with the summer maximum concentrated over the Pindus range, Thessaly and the central highlands⁷. The resistivity present at the instant lightning occurs is therefore systematically different from the annual mean used in design. Backflashover of Hellenic 150 kV lines has previously been assessed with constant-parameter footings and a single annual resistivity¹⁹.

The research gap is thus twofold: no published study evaluates the two dispersive mechanisms of the footing simultaneously, and none weights the soil resistivity by the season in which lightning actually occurs. The contributions are: (i) a coupled FD-plus-ionization footing model, verified against both the CIGRE/Weck ionization curve and an independent parameterisation of the Alipio–Visacro soil model, with a map of the error committed by each common simplification; (ii) a demonstration that the two mechanisms are very nearly separable, with the physical reason why; (iii) a lightning-weighted seasonal treatment of ρ_0 and its effect on BFR; and (iv) revised effective-length design curves for the standard supplementary counterpoise.

The remainder of the paper is organised as follows. Section II defines the test system. Section III presents the materials and methods: the soil model, the electrode and network models, the ionization treatment, the flashover criterion and the verification of the whole chain. Section IV reports the results and discussion, and Section V concludes.

II. TEST SYSTEM

The line is a heavy double-circuit 150 kV line of the Hellenic transmission system. Phase conductors are ACSR GROSBEAK 636 kcmil of 25.2 mm outside diameter; a single 9.5 mm seven-strand galvanised steel shield wire is carried at the tower peak. Insulation is provided by strings of ten cap-and-pin discs of 146.05 mm spacing without arcing horns, so the arcing distance is 1.4605 m, which is the gap used in the flashover model, while the complete suspension assembly including end fittings measures 1860 mm and locates the conductor below the crossarm. The mean span is 340 m, so the 42 km route comprises approximately 124 spans. Sags of 9.0 m (phase) and 6.0 m (shield wire) are assumed; mean conductor heights, taken as the attachment height less two thirds of the sag, are then 13.95, 17.85 and 21.75 m for the bottom, middle and top phases and 29.01 m for the shield wire. These enter the coupling factors and the incidence calculation; the sag is the one geometric input not fixed by the drawings.

Tower geometry was extracted from the standard single-line construction drawings for the S4 suspension tower and is given in Table no 1 and Figure no 1. The vertical dimension chain from ground level is 4210 + 17600 + 3900 + 3900 + 3400 mm, totalling 33 010 mm. Subtracting the 1860 mm suspension string from the lowest crossarm at 21 810 mm gives an attachment height of 19 950 mm, reproducing exactly the suspension height quoted independently in the tower specification, which confirms the extraction. Leg extensions of 0 and +1.5 m are typical on the route and the nominal tower is analysed here.

Element	Height above ground (m)	Half-spacing from axis (m)
Shield wire (peak)	33.01	0
Top crossarm / conductor	29.61 / 27.75	± 3.15
Middle crossarm / conductor	25.71 / 23.85	± 5.15
Bottom crossarm / conductor	21.81 / 19.95	± 4.05
Base width (square section)	0	5.775 (face width)

Table no 1: S4 suspension tower geometry, from the standard single-line construction drawings of the tower series

The footing is likewise taken from the standard specification. Every tower is supplied with four hot-dip galvanised steel rods of 20 mm diameter and 2.0 m length, one bonded to each of the four independent concrete foundations. Where the resulting resistance is inadequate, supplementary grounding is provided by a single-strand galvanised steel counterpoise of 10 mm diameter connected to the tower at the first bolt above ground; up to four radials are used, at a burial depth of 0.8 m. Table no 2 gives the computed low-frequency resistance of both arrangements. In karst at 2000 $\Omega \cdot m$ the rods alone give 283 Ω , which is why supplementary counterpoise is in practice mandatory on this terrain; four 30 m radials reduce this to 32 Ω , and this is adopted as the base-case footing throughout.

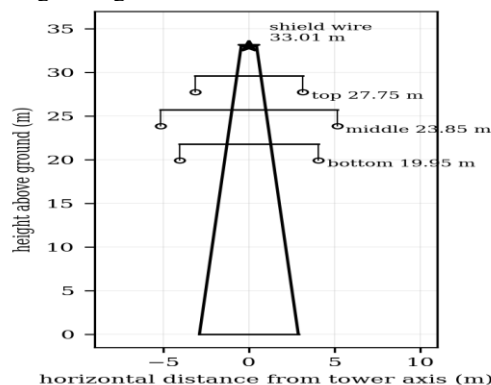


Figure no 1: S4 double-circuit suspension tower of the Hellenic 150 kV system, with the single shield wire at the peak. Open circles mark the conductor attachment points below the 1.86 m suspension strings.

ρ_0 ($\Omega \cdot m$)	100	300	700	1000	2000	3000	5000	10000
4×2 m rods only	14.1	42.4	99.0	141.4	282.7	424.1	706.8	1413.6
+ 4×30 m counterpoise	1.6	4.9	11.3	16.2	32.3	48.5	80.9	161.7

Table no 2: Computed low-frequency footing resistance (Ω).

III. MATERIALS AND METHODS

3.1 Frequency-Dependent Soil Parameters

The Alipio–Visacro model³, adopted by CIGRE WG C4.33⁴, is used:

$$\sigma(f) = \sigma_0 + \sigma_0 \cdot h(\sigma_0) \cdot (f \cdot 10^{-6} - 0.1)^{\gamma} \quad [\text{mS/m}], \quad h(\sigma_0) = H \cdot \sigma_0^{(-0.73)}, \quad \gamma = 0.54 \quad (1)$$

$$\epsilon_r(f) = 7.6 \cdot 10^3 \cdot f^{(-0.4)} + 1.3 \quad (2)$$

with f in Hz and σ_0 in mS/m. The coefficient H takes the values 0.95, 1.26 and 1.58 for the relatively conservative, mean and relatively low-impedance levels of the fit; $H = 1.26$ is used unless stated. The expression is clipped to $\sigma = \sigma_0$ below 100 kHz, where the bracketed term is non-positive.

The implementation was checked against the independent Visacro–Alipio parameterisation $\sigma(f) = \sigma + 4.68 \cdot 10^{-6} \sigma^{0.27} f^{0.54}$ ²¹. At $\rho_0 = 1000 \Omega \cdot m$ and 1 MHz the two forms give $\rho(f)/\rho_0 = 0.457$ and 0.443, a 3.2 % difference. The agreement is not uniform: across 300–5000 $\Omega \cdot m$ the forms differ by 1 % or less above 2 MHz but by up to 20 % at 200 kHz and 5000 $\Omega \cdot m$, where the offset term of Eq. (1) dominates. That spread measures the uncertainty in the empirical soil fit itself and is comparable to some of the effects computed below. Figure no 2 shows the resulting dispersion. A feature often passed over is that the model predicts essentially no dispersion below 100 kHz; the consequences for a median first stroke are examined in Section 4.3.

3.2 Electrode Harmonic Impedance

Each electrode is represented by Sunde's transmission-line model⁸. For a geometric factor Λ the per-unit-length parameters are taken as the self-consistent pair

$$Y' = kg \cdot \pi \cdot (\sigma + j\omega\epsilon)/\Lambda, \quad Z' = j\omega\mu_0 \cdot \Lambda/(kg \cdot \pi), \quad \text{so that} \quad Z'Y' = \gamma_s^2 \quad (3)$$

with $kg = 2$ for a vertical rod, for which $\Lambda = \ln(4L/a) - 1$ recovers Dwight's resistance⁹, and $kg = 1$ for a horizontal conductor buried close to the surface, for which $\Lambda = \ln(2L/\sqrt{(2ad)}) - 1$ recovers Sunde's. The input impedance of an open-ended electrode of length L is $Z_c \cdot \coth(\gamma_s L)$ with $Z_c = \sqrt{Z'/Y'}$. Rod and counterpoise are combined in parallel at each leg, the four legs are paralleled, and a utilisation factor of 0.80 accounts for mutual coupling between them. The transverse coupling a full-wave or method-of-moments treatment resolves¹⁵ is not captured; see Section 4.8.

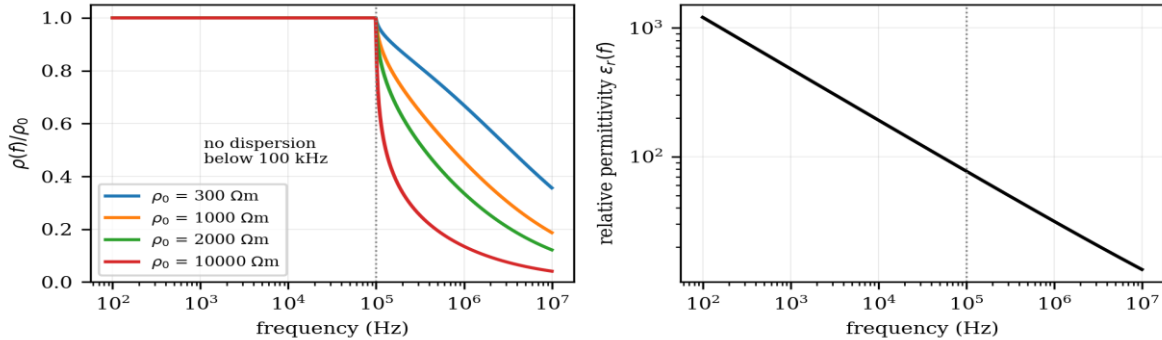


Figure no 2: Alipio–Visacro soil model: (left) resistivity relative to its low-frequency value; (right) relative permittivity.

3.3 Soil Ionization

Ionization is represented as a contraction of the geometric factor, $\Lambda \rightarrow q\Lambda$ with $q \leq 1$, where q is calibrated so that in the constant-parameter limit the electrode reproduces the CIGRE/Weck dynamic resistance^{1,10} exactly:

$$RW(I) = R_0 / \sqrt{(1 + I/I_g)}, \quad I_g = \rho_0 \cdot E_0 / (2\pi R_0^2), \quad q = RW(I)/R_0 \quad (4)$$

The critical field E_0 is taken as 400 kV/m following CIGRE, with $E_0 = 241 \cdot \rho_0^{0.215}$ kV/m after Oettl¹¹ retained as a sensitivity case. The ionized geometry is then evaluated with the frequency-dependent parameters of Section 3.1, which is what makes the model coupled. Because the injected waveform is prescribed, q is obtained by fixed-point iteration on the peak footing current, converging in three to five iterations. The resulting reduction is the quantity described elsewhere as the impulse efficiency of the electrode²⁰.

Four footing representations are compared throughout: G0, constant soil parameters without ionization (conventional practice); G1, frequency-dependent soil without ionization (CIGRE TB 781 practice); G2, constant soil parameters with ionization (conventional IEEE practice); and G3, coupled frequency-dependent soil with ionization, proposed here.

3.4 Line, Tower and Insulation Models

The struck tower is a lossless line of surge impedance $ZT = 187.9 \Omega$, from the conical approximation with a 2.89 m base radius, and travel time 130 ns at 0.85c. Damping follows Ishii's multistory representation¹² as a parallel R–L element in series with the tower line, $RD = ZT(1 - k)/k$ with $k = 0.75$ and $LD = 2RD\tau$. The parallel combination vanishes at power frequency and so introduces no spurious d.c. resistance, a defect a uniformly attenuating line would possess. Spans of 340 m are represented on each side of the struck tower. Rather than truncate the line, the ladder of adjacent towers is iterated to its fixed point, so the network represents an effectively infinite line and carries no truncation error; convergence is reached by about 24 spans. Span damping uses the same parallel R–L structure, attenuating to 0.85 per span at high frequency, together with the 3.3 Ω/km d.c. resistance of the steel shield wire, which is what allows the ladder to converge at low frequency. The shield-wire surge impedance is 488.9 Ω including corona, and the coupling factors to the bottom, middle and top phases are 0.125, 0.165 and 0.228. The linear network is solved in the frequency domain by chain matrices and transformed with a 32 768-point FFT over a 328 μs window, long enough that the 77.5 μs tail decays within it; a raised-cosine taper over the last fifth suppresses wrap-around.

The stroke is a Heidler current of median first-stroke shape, 3.83 μs front and 77.5 μs time to half value¹³. Insulator flashover is determined by the leader progression model¹⁷,

$$dI/dt = KL \cdot u(t) \cdot [u(t)/(d - l) - E_{l0}], \quad KL = 1.3 \cdot 10^{-6} \text{ m}^2/(\text{V}^2\text{s}), \quad E_{l0} = 585 \text{ kV/m} \quad (5)$$

with flashover declared when the leader bridges the 1.4605 m gap. The leader model is preferred to a volt–time curve because the coupled footing model alters the waveshape and not only the peak of the insulator voltage, and because a volt–time criterion spuriously registers flashover on the sub-microsecond tower-reflection spike, which carries far too little charge to propagate a leader. A power-frequency voltage of half the peak phase-to-earth value is superimposed. All three phases are examined and the lowest critical current is reported.

3.5 Backflashover Rate

The critical current I_c is found by bisection on the stroke amplitude. Incidence follows the IEEE Std 1243 form of Anderson's expression^{2,22}, $NL = Ng(28h^{0.6} + b)/10$ with h the mean shield-wire height of 29.01 m, giving $NL = 84.5$ flashes per 100 km per year. The ground flash density is taken as $Ng = 4$ flashes/ km^2/yr , consistent with the mapped activity over continental Sterea Ellada^{6,7} but adopted as an assumption rather than a site measurement; every rate below scales linearly with it, so the ratios between footing models, which are the results of interest, are unaffected. The BFR follows as $0.6 \cdot NL \cdot P(I > I_c)$ with the CIGRE log-normal first-stroke distribution, median 31.1 kA and log-standard deviation 0.484. Electrogeometric treatments of incidence¹⁸ would refine the absolute rates but, every result being a ratio between models sharing the same incidence, not the conclusions.

3.6 Model Verification and Numerical Convergence

Two independent verifications were performed before any result was computed. The ionization representation reproduces the CIGRE/Weck curve to within 0.0 % over 500–5000 $\Omega \cdot \text{m}$ and 5–100 kA (Figure no 3); this follows from the calibration of Eq. (4) but confirms that the transmission-line electrode returns the intended low-frequency limit. The soil model agrees with an independent published parameterisation as noted in Section 3.1. As a third check, the FD-only variant G1 reduces the BFR relative to constant soil by 16.4 %, 47.1 % and 26.4 % at 300, 1000 and 2000 $\Omega \cdot \text{m}$, against approximately 8 %, 20 % and 25 % reported by Visacro and Silveira for a 138 kV line⁵; the direction and order of magnitude agree, the larger values here being attributable to the compact four-rod-plus-counterpoise footing, which is more strongly dispersive than the extended counterpoise arrangements of the reference case.

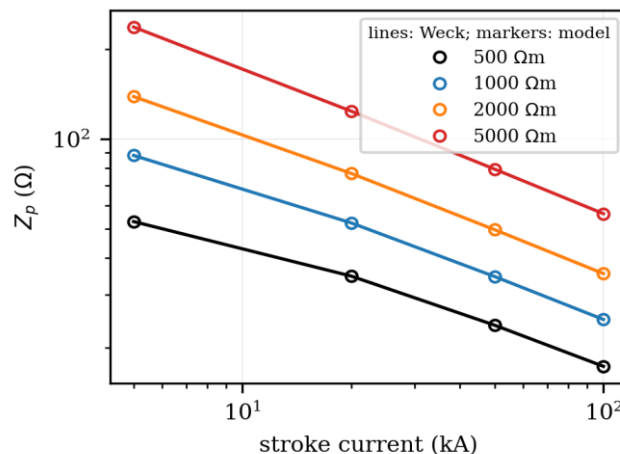


Figure no 3: Verification of the ionization representation against the CIGRE/Weck dynamic resistance. Lines: Weck; markers: present model.

Numerical convergence was confirmed by refinement. Halving the time step from 20 ns to 2.5 ns at a fixed 328 μ s window changes the critical current by less than 0.02 %; the leader integration is insensitive to its step over the same range; extending the ladder beyond 24 spans changes the BFR by less than 0.001 %; and the bisection resolves I_c to 12 A. The fixed-point iteration for the ionized state converges monotonically and uniquely, q varying by less than 0.21 % beyond three iterations at the most strongly ionized condition examined. The Heidler time constants are obtained numerically for each requested front time and time to half value, so the waveform reproduces the IEC virtual front time to better than 0.1 %; the customary substitutions $\tau_1 = t_f/2.1$ and $\tau_2 = t_h/0.69$ are in error by 25 % on the front at the median first stroke and by 70 % for a 0.2 μ s front, which would otherwise bias every front-time-dependent result. As an independent check on the network solution, the zero-frequency gain of the insulator-voltage transfer function agrees with the ratio of the time integrals of the computed voltage and the injected current to 0.01 %. The electrode reproduces the classical Dwight and Sunde closed-form resistances exactly in the d.c. limit, and both footing and network are passive at every frequency.

IV. RESULT AND DISCUSSION

4.1 Impulse Impedance and the Cost of Each Simplification

Figure no 4 maps, over low-frequency resistivity and stroke current, the percentage by which each simplified footing representation overestimates the impulse impedance Z_p relative to the coupled model G3, for the base-case footing at a median 3.83 μ s front.

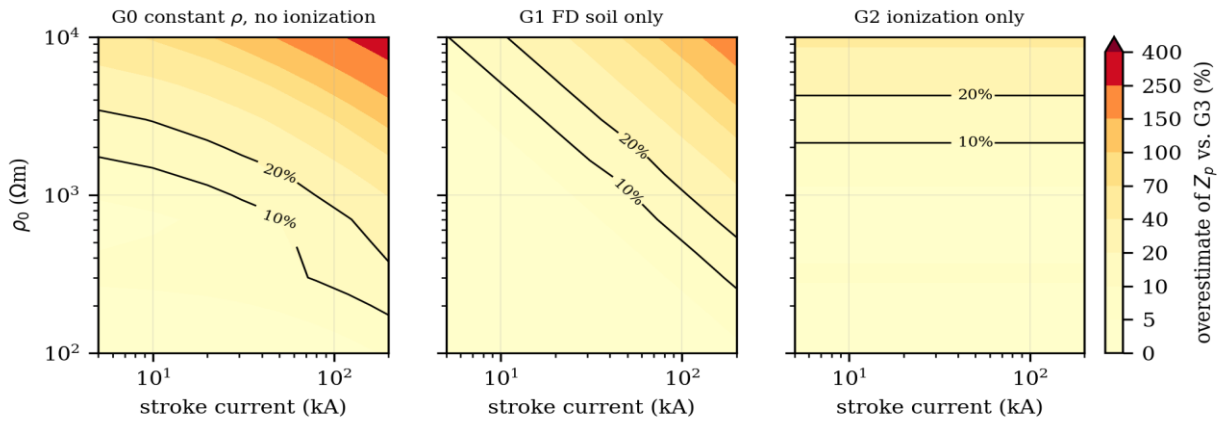


Figure no 4: Overestimate of the footing impulse impedance Z_p committed by each simplified representation, relative to the coupled model G3. Base-case footing, $t_f = 3.83 \mu$ s. Black contours mark the 10 % and 20 % error levels.

The structure of the three panels is instructive. The constant-parameter error (G0) grows in both variables and reaches 344 %, its contours running diagonally because both mechanisms contribute. The FD-only error (G1), which omits ionization, reaches 204 %; it grows with current, as expected, but also with resistivity. The latter follows from the onset current of Eq. (4): since R_0 is itself proportional to ρ_0 , the combination $I_g = \rho_0 E_0 / (2\pi R_0^2)$ varies as $1/\rho_0$, and the computed values confirm this exactly, falling from 811 kA at 300 $\Omega \cdot m$ to 24 kA at 10 000 $\Omega \cdot m$. A given stroke therefore drives the electrode progressively further into ionization as the soil becomes more resistive, and the penalty for omitting ionization grows accordingly. The ionization-only error (G2), which omits soil dispersion, is the one clean case: its contours are exactly horizontal, so the error depends on resistivity alone and not at all on current. That independence is a direct consequence of the separability established in Section 4.2, and it means the cost of neglecting frequency dependence can be tabulated against ρ_0 as a single column. It reaches 46 % at 10 000 $\Omega \cdot m$.

It is worth comparing the G2 column against the CIGRE WG C4.33 thresholds. At the electrode level the agreement is poor: the error of neglecting frequency dependence is only 2.9 % at the 700 $\Omega \cdot m$ level above which CIGRE declares the refinement mandatory, and does not reach 10 % until approximately 2100 $\Omega \cdot m$. Judged on impulse impedance alone the guidance looks markedly conservative; judged at system level it does not, because the corresponding error in backflashover rate at 700 $\Omega \cdot m$ is 70 % (Section 4.4). The exceedance probability of the stroke-current distribution amplifies a modest impedance error into a large rate error, so the threshold is well placed, but only when read at the level at which it matters. This gap between electrode-level and system-level error is the most easily overlooked result in this paper.

4.2 Separability of the Two Mechanisms

A natural hypothesis is that the two mechanisms interact strongly. They do not. Writing the three ratios relative to G0, the residual

$$\Delta = Z_p(G3)/Z_p(G0) - [Z_p(G1)/Z_p(G0)] \cdot [Z_p(G2)/Z_p(G0)] \quad (6)$$

is smaller than 10^{-13} percentage points everywhere on the grid of Figure no 4. The reason is structural. Ionization enters the electrode model only through the geometric factor Λ , and scaling Λ scales Z' and Y' inversely, so the propagation

constant $\gamma_s = \sqrt{(Z/Y')}$ is untouched and only the characteristic impedance $Z_c = \sqrt{(Z'/Y')}$ changes. Frequency dependence, by contrast, enters only through σ and ϵ and therefore acts on γ_s . The two mechanisms operate on orthogonal factors of the same transmission-line solution and multiply exactly.

This exactness is a property of the representation rather than of nature, and must therefore be tested against a formulation that does not build it in. A second ionization model was implemented in which the rod and the counterpoise ionize independently, each according to the current it carries, with a cylindrical ionization boundary at $r = \rho_0 I / (2\pi \epsilon_0 L)$; the two geometric factors are then free to move apart, and the division of current between rod and counterpoise shifts as ionization develops. Departures from exact separability do appear, but over 144 combinations of resistivity, current and electrode configuration they reach only 0.25 percentage points, with a mean of 0.012 percentage points. The separability is therefore a property of the physics and not merely of the algebra. The practical conclusion is useful: the two effects may be applied as independent multiplicative corrections, and no coupled solution is needed provided both are applied. What is not permissible is to apply only one of them.

4.3 Front-Time Dependence

Because the Alipio–Visacro model predicts no dispersion below 100 kHz, the benefit of the FD representation depends strongly on the steepness of the front. Figure no 5 quantifies this. At 300 $\Omega \cdot m$ the FD correction to Z_p falls from -27.6% for a 0.2 μs front to -5.1% at the median first-stroke front of 3.83 μs and -0.9% at 10 μs . At 10 000 $\Omega \cdot m$ the correction is large at all front times, from -34.1% to -28.5% , because the dispersion is by then severe enough to matter even for the modest high-frequency content of a slow front.

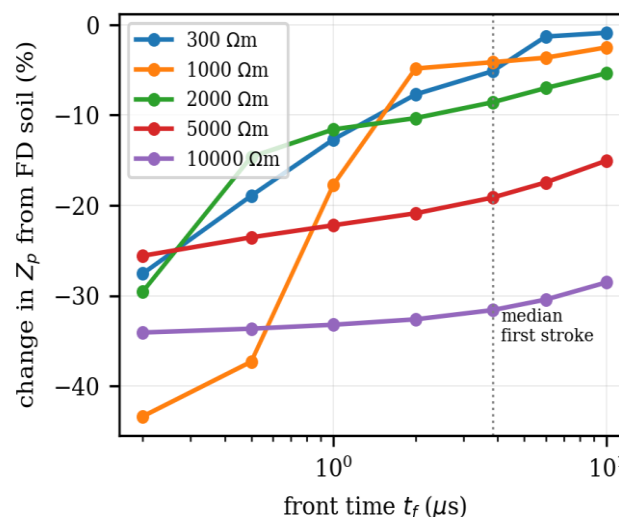


Figure no 5: Dependence of the frequency-dependent soil correction to Z_p on stroke front time.

The practical reading is that for moderate resistivity the FD refinement matters chiefly for subsequent strokes and for the fast tower-reflection content of the transient, whereas for the high-resistivity karst of concern here it matters for first strokes too, exceeding 19 % at 5000 $\Omega \cdot m$ even at the median front. This also explains why the conservativeness coefficient H of Eq. (1) had almost no influence on Z_p at the median front: at 100 kHz and below, all three levels of the fit coincide.

4.4 Critical Current and Backflashover Rate

Table no 3 and Figure no 6 give the critical current and the resulting BFR for the four representations; Figure no 7 shows representative insulator voltages. The percentage errors quoted below are computed from Table no 3.

ρ_0 ($\Omega \cdot m$)	G0	G1	G2	G3
100	123.3 / 0.11	124.1 / 0.11	123.7 / 0.11	124.5 / 0.11
300	104.9 / 0.30	108.1 / 0.25	106.6 / 0.28	109.8 / 0.23
700	80.7 / 1.24	89.0 / 0.76	84.7 / 0.98	93.7 / 0.57
1000	67.3 / 2.81	77.6 / 1.49	72.3 / 2.06	84.5 / 0.99
2000	40.9 / 14.51	45.9 / 10.68	45.1 / 11.24	51.9 / 7.36
3000	29.8 / 27.18	34.8 / 20.66	32.9 / 23.04	39.4 / 15.80
5000	20.4 / 40.97	26.0 / 32.67	22.5 / 37.98	30.0 / 26.81
10000	13.1 / 48.80	20.2 / 41.28	14.4 / 47.88	24.6 / 34.72

Table no 3: Critical current I_c (kA) and backflashover rate BFR (flashovers per 100 km per year) for the base-case footing at $N_g = 4$ flashes/km²/yr. Each cell gives I_c / BFR.

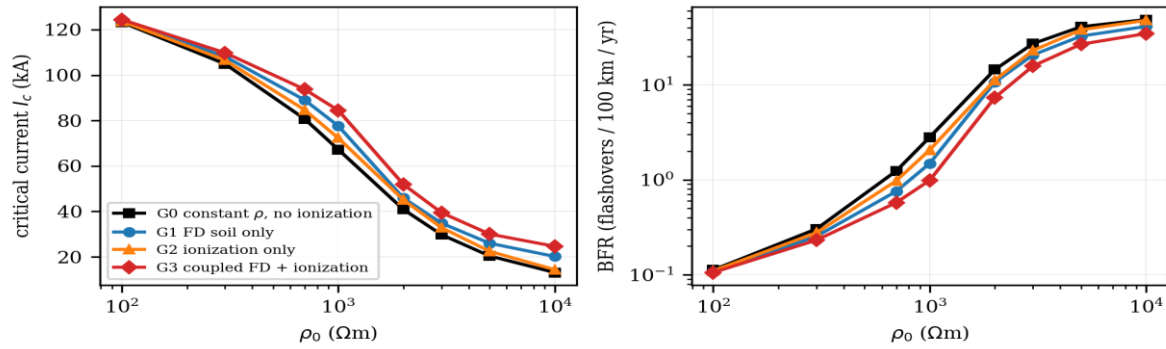


Figure no 6: Critical current (left) and backflashover rate (right) against low-frequency resistivity for the four footing representations.

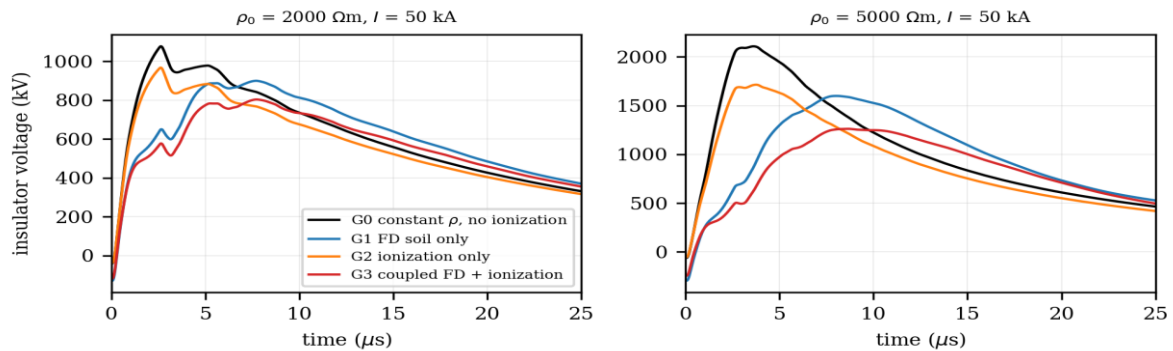


Figure no 7: Insulator voltage on the most stressed phase for a 50 kA first stroke, base-case footing.

The errors peak not at the highest resistivity but at intermediate values, around 1000 $\Omega \cdot m$, where the constant-parameter model overestimates BFR by 185 %, the FD-only model by 51 % and the ionization-only model by 109 %. This is a consequence of the probability transform rather than the electromagnetics: near 1000 $\Omega \cdot m$ the critical current falls close to the 31.1 kA median of the stroke-current distribution, where the exceedance probability is steepest and a given error in I_c produces the largest error in BFR. At 100 $\Omega \cdot m$ all four models agree to within 7 %, the footing no longer being the controlling element, and above 5000 $\Omega \cdot m$ the errors decline again as the exceedance probability saturates.

Two engineering observations follow. First, the line studied here would have an acceptable computed BFR of 0.99 per 100 km per year only if ρ_0 could be held near 1000 $\Omega \cdot m$ with the standard 30 m counterpoise; in the 2000–5000 $\Omega \cdot m$ range typical of dry karst the rate rises to between 7.4 and 26.8 even with the coupled model, which for a 42 km route implies between 3.1 and 11.3 backflashovers per year. Second, the conventional calculation would have placed those figures at 14.5 and 41.0, so the coupled model roughly halves the predicted exposure and would materially change a decision on remedial counterpoise or surge arresters.

4.5 Effective Counterpoise Length

The classical effective-length expressions¹⁶ were derived for constant soil parameters. Under frequency-dependent soil the high-frequency attenuation along a buried conductor is lower than the constant- σ model predicts, so the length beyond which further counterpoise ceases to help is longer. Figure no 8 and Table no 4 give L_{eff} , defined as the shortest radial length within 5 % of the asymptotic minimum impulse impedance, for the standard 10 mm counterpoise.

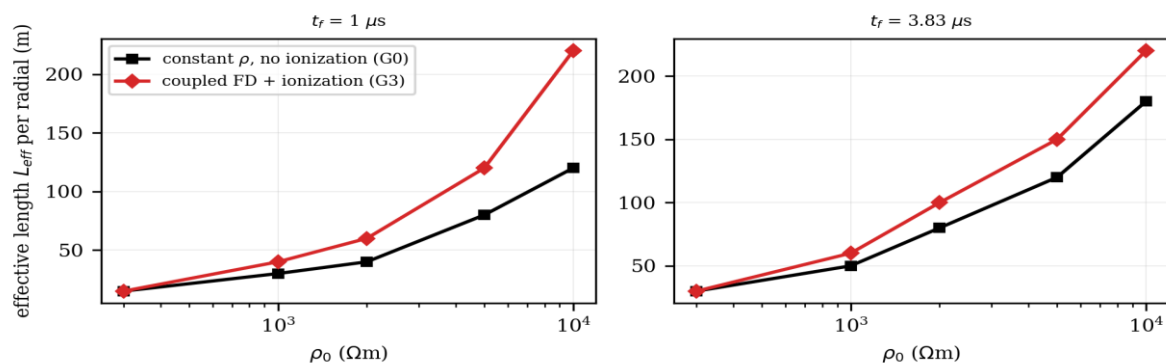


Figure no 8: Effective length per counterpoise radial under the conventional and the coupled models.

Case	300	1000	2000	5000	10000
tf = 1 μ s, conventional (G0)	15	30	40	80	120
tf = 1 μ s, coupled (G3)	15	40	60	120	220
tf = 3.83 μ s, conventional (G0)	30	50	80	120	180
tf = 3.83 μ s, coupled (G3)	30	60	100	150	220

Table no 4: Effective length per radial, L_{eff} (m).

The correction is negligible below about 700 $\Omega \cdot m$ but substantial above it: at 5000 $\Omega \cdot m$ and a 1 μ s front the effective length rises from 80 m to 120 m, and at 10 000 $\Omega \cdot m$ from 120 m to at least 220 m. Design practice that stops the counterpoise at the conventional effective length therefore forgoes real available improvement on precisely the high-resistivity spans where it is most needed. At 300 $\Omega \cdot m$ the conventional curve is adequate, consistent with the CIGRE threshold. The 220 m entries are lower bounds set by the longest radial examined.

4.6 Seasonal Weighting of Resistivity

Seasonal resistivity is derived from an Archie-type relation $\rho \propto \theta^{-(2)}^{14}$ applied to representative volumetric water contents for thin karst soils of 0.30, 0.24, 0.12 and 0.20 in winter, spring, summer and autumn. Relative to the annual mean this gives factors of 0.36, 0.57, 2.26 and 0.81. Two lightning weightings are considered. The national weighting uses the measured shares of 33.8 % summer and 43.5 % autumn, with the remaining 22.7 % assigned to the cold half-year⁶; because that residual is not resolved into spring and winter in the source, the two are combined into a single wet-season block. The inland weighting, offered as a scenario rather than a measurement, raises the summer share to 55 % to reflect the dominance of convective activity over the continental highlands⁷. Figure no 9 shows the seasonal resistivity factors alongside the observed distribution of flashes, and Table no 5 gives the resulting rates.

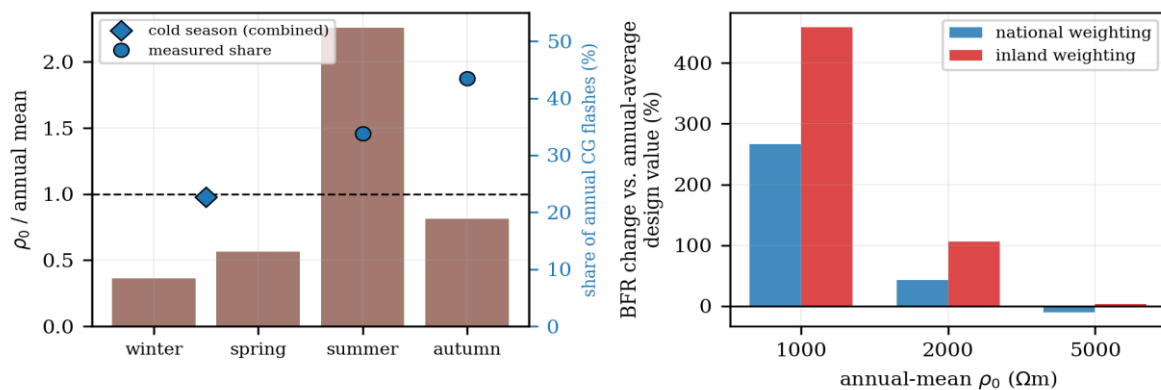


Figure no 9: (Left) seasonal resistivity factors and the seasonal distribution of cloud-to-ground flashes. (Right) change in BFR when resistivity is weighted by lightning incidence rather than averaged annually.

Annual-mean ρ_0 ($\Omega \cdot m$)	Annual average	National weighting	Change	Inland weighting	Change
1000	0.99	3.62	+267 %	5.52	+459 %
2000	7.36	10.52	+43 %	15.14	+106 %
5000	26.81	24.12	-10 %	27.77	+4 %

Table no 5: Backflashover rate computed with an annual-average resistivity and with a lightning-weighted seasonal resistivity (flashovers per 100 km per year, coupled model G3).

The lightning-weighted mean resistivity exceeds the annual mean by 22 % under the national weighting and by 56 % under the inland weighting. But the effect on BFR is far larger than that modest shift suggests: +267 % and +459 % respectively at an annual mean of 1000 $\Omega \cdot m$. The mechanism is convexity. BFR is a strongly convex function of ρ_0 over this range, so by Jensen's inequality the expectation of BFR across the seasons exceeds the BFR evaluated at the expected resistivity. Averaging the resistivity first and computing the BFR second, which is what conventional design does, is therefore not merely imprecise but systematically non-conservative. At 5000 $\Omega \cdot m$ the effect reverses slightly, to -10.0 % under the national weighting, because BFR has saturated against its ceiling and the function is locally concave.

The magnitude of these figures should be read with the status of their inputs in mind. The lightning shares are measured; the resistivity factors are a physically-motivated scenario rather than a measurement, and a monitoring campaign on the route would be required to fix them. The qualitative conclusion does not depend on their precise values: any seasonal variation of ρ_0 at all, combined with the convexity of BFR, biases the conventional calculation in the non-conservative direction.

4.7 Sensitivity to the Critical Field

Figure no 10 compares the CIGRE constant critical field with Oettlé's resistivity-dependent expression. The two diverge with resistivity, from 0.2 % at 100 $\Omega \cdot m$ to 32.7 % at 10 000 $\Omega \cdot m$. This uncertainty in E_0 is of the same order as the entire FD correction at moderate resistivity and exceeds it at low resistivity: the critical field for ionization is at present the less well determined of the two mechanisms. By contrast the conservativeness coefficient H produced a spread below 0.2 % at the median front time, for the reason given in Section 4.3, though its influence would be greater for subsequent-stroke fronts.

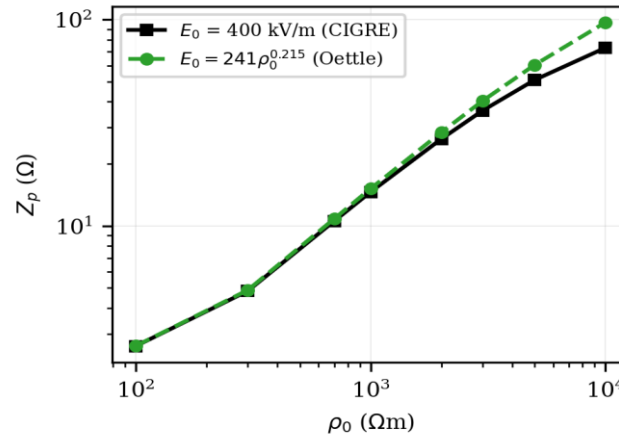


Figure no 10: Sensitivity of the coupled impulse impedance to the assumed critical field for ionization.

4.8 Limitations

The results rest on a frequency-domain solution of a linear network with a quasi-linearised ionization, not on a full electromagnetic transient simulation. Three limitations follow. First, the ionization treatment is quasi-static: it captures the reduction of footing impedance at the current peak but not the hysteresis between the rising and falling parts of the wave, nor the finite time constant of ionization itself. Confirmation with a time-domain solver, in which a nonlinear element sits within a vector-fitted rational approximation of the frequency-dependent electrode impedance, would be valuable, and the present results provide the reference against which such a model can be checked. Second, the transmission-line electrode neglects transverse electromagnetic coupling, and the mutual coupling between the four legs is a single scalar utilisation factor. Third, no field measurements were available, neither soil resistivity on the route nor recorded outages, so the study is comparative by construction: every headline result is a difference between models evaluated on identical assumptions, and absolute BFR figures are given only to make those differences concrete.

Surge arresters were excluded from scope but are the natural extension, and the results suggest a specific unexploited question: arrester energy duty is governed by the tail of the ground current, and Figure no 7 shows that the coupled model alters the tail differently from the peak, so a representation adequate for backflashover may be inadequate for arrester energy absorption.

V. CONCLUSION

The transient response of the standard footing of the Hellenic 150 kV Series-4 tower has been computed with frequency-dependent soil parameters and soil ionization represented together, and propagated to the backflashover rate of a 42 km line in the karst of Sterea Ellada. Omitting either dispersive mechanism is a first-order error: a constant-resistance footing overestimates the impulse impedance by up to 344 % and the backflashover rate by up to 185 %, while frequency dependence alone still overestimates BFR by up to 51 %, and ionization alone by up to 109 %. The largest errors occur near 1000 $\Omega \cdot m$, where the critical current coincides with the median stroke current.

The two mechanisms are, to an excellent approximation, multiplicatively separable, because ionization acts on the geometric factor of the electrode and hence on its characteristic impedance, while frequency dependence acts on the propagation constant of the soil. Two independent correction factors therefore suffice; what is not permissible is to apply only one. Because the soil model predicts no dispersion below 100 kHz, the value of the frequency-dependent refinement is strongly front-time dependent: at 300 $\Omega \cdot m$ it is worth 27.6 % for a 0.2 μs front but only 5.1 % at the median first-stroke front, whereas above 5000 $\Omega \cdot m$ it exceeds 15 % at all front times.

Because BFR is convex in low-frequency resistivity, weighting seasonal resistivity by the observed bimodal Greek lightning climatology raises the computed rate by 43 % to 267 % at moderate resistivity under national weighting and by up to 459 % under an inland-convective weighting, so averaging resistivity before computing BFR is systematically non-conservative. The effective length of the standard 10 mm counterpoise is longer than the conventional calculation indicates once frequency dependence is included, 120 m rather than 80 m at 5000 $\Omega \cdot m$ for a 1 μs front, so available mitigation is being forgone on exactly the spans where it is most needed. Finally, the uncertainty in the critical field for ionization reaches 32.7 % at 10 000 $\Omega \cdot m$, comparable to or larger than the frequency-dependence correction itself; better determination of E_0 in high-resistivity soils would repay more than further refinement of the soil dispersion model.

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